

# EKUIVALEN LOGIS

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# PENGANTAR

- **Tautologi** pasti ekuivalen secara logis
- **Kontradiksi** pasti ekuivalen secara logis
- How about **contingent**??



# CONTOH 1

- Dewi sangat cantik dan peramah
- Dewi peramah dan sangat cantik

A = Dewi sangat cantik

B = Dewi peramah

○  $A \wedge B$

○  $B \wedge A$

○  $(A \wedge B) \equiv (B \wedge A)$

A	B	$A \wedge B$	$B \wedge A$
F	F	F	F
F	T	F	F
T	F	F	F
T	T	T	T



## CONTOH 2

- Badu tidak pandai, atau dia tidak jujur
- Adalah tidak benar jika Badu pandai dan jujur

- $A$  = Badu pandai
- $B$  = Badu jujur



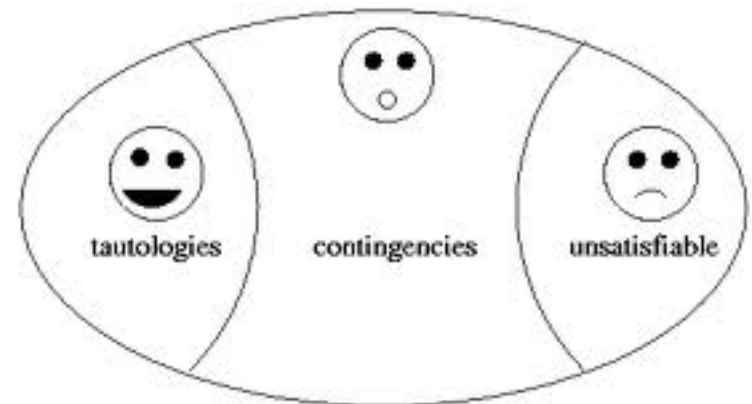
- $\neg A \vee \neg B$
- $\neg(A \wedge B)$

A	B	$\neg A$	$\neg B$	$A \wedge B$	$\neg A \vee \neg B$	$\neg(A \wedge B)$
F	F	T	T	F	T	T
F	T	T	F	F	T	T
T	F	F	T	F	T	T
T	T	F	F	T	F	F



- Baru dapat dikatakan ekuivalensi jika dihubungkan dengan perangkat ekuivalensi dan menghasilkan tautologi
- $\neg A \vee \neg B \leftrightarrow \neg(A \wedge B)$

$\neg A \vee \neg B$	$\neg(A \wedge B)$	$\neg A \vee \neg B \leftrightarrow \neg(A \wedge B)$
T	T	T
T	T	T
T	T	T
F	F	T



# KOMUTATIF

- $(A \wedge B) \equiv (B \wedge A)$
- $(A \vee B) \equiv (B \vee A)$
- $(A \leftrightarrow B) \equiv (B \leftrightarrow A)$
- **“ $\rightarrow$ ” tidak memiliki sifat komutatif**
- $(A \rightarrow B)$  dengan  $(B \rightarrow A)$  memiliki nilai kebenaran yang berbeda

A	B	$A \rightarrow B$	$B \rightarrow A$
F	F	T	T
F	T	T	F
T	F	F	T
T	T	T	T



# ASOSIATIF

- $((A \wedge B) \wedge C) \equiv (A \wedge (B \wedge C))$
- Berlaku pula untuk “ $\vee$ ” dan “ $\leftrightarrow$ ”
- Tidak berlaku untuk “ $\rightarrow$ ”

A	B	C	$A \wedge B$	$(A \wedge B) \wedge C$	$B \wedge C$	$A \wedge (B \wedge C)$
F	F	F	F	F	F	F
F	F	T	F	F	F	F
F	T	F	F	F	F	F
F	T	T	F	F	T	F
T	F	F	F	F	F	F
T	F	T	F	F	F	F
T	T	F	T	F	F	F
T	T	T	T	T	T	T



# TIDAK BERLAKU UNTUK PERANGKAI YANG BERBEDA..!!

- $((A \wedge B) \vee C)$  dan  $(A \wedge (B \vee C))$

A	B	C	$A \wedge B$	$(A \wedge B) \vee C$	$B \vee C$	$A \wedge (B \vee C)$
F	F	F	F	<b>F</b>	F	<b>F</b>
F	F	T	F	<b>T</b>	T	<b>F</b>
F	T	F	F	<b>F</b>	T	<b>F</b>
F	T	T	F	<b>T</b>	T	<b>F</b>
T	F	F	F	<b>F</b>	F	<b>F</b>
T	F	T	F	<b>T</b>	T	<b>T</b>
T	T	F	T	<b>T</b>	T	<b>T</b>
T	T	T	T	<b>T</b>	T	<b>T</b>





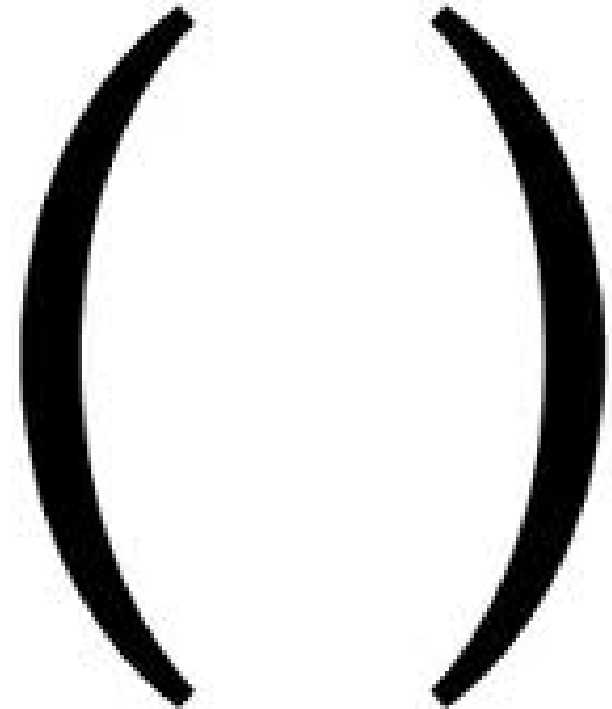
# PARENTHESES

$$(\neg A \vee \neg B) \wedge A \wedge C$$

$$\equiv A \wedge (\neg A \vee \neg B) \wedge C$$

$$\equiv (A \wedge (\neg A \vee \neg B)) \wedge C$$

komutatif  
parentheses



# HUKUM-HUKUM LOGIKA

- Jika anda tidak belajar maka anda gagal
- Anda harus belajar atau anda akan gagal
- $A =$  anda tidak belajar
- $B =$  anda gagal

○  $A \rightarrow B$

○  $\neg A \vee B$

A	B	$\neg A$	$A \rightarrow B$	$\neg A \vee B$
F	F	T	T	T
F	T	T	T	T
T	F	F	F	F
T	T	F	T	T

$$A \rightarrow B \equiv \neg A \vee B$$



# DE MORGAN'S LAW

- $\neg(A \wedge B) \equiv \neg A \vee \neg B$
- $\neg(A \vee B) \equiv \neg A \wedge \neg B$

## Contoh

Jika Badu tidak sekolah maka Badu tidak akan pandai

Jika Badu pandai maka Badu pasti sekolah

A = Badu sekolah

B = Badu pandai

- $\neg A \rightarrow \neg B$
- $B \rightarrow A$



$\neg A \rightarrow \neg B$

$B \rightarrow A$

A	B	$\neg A$	$\neg B$	$\neg A \rightarrow \neg B$	$B \rightarrow A$
F	F	T	T	T	T
F	T	T	F	F	F
T	F	F	T	T	T
T	T	F	F	T	T

$\neg A \rightarrow \neg B \equiv B \rightarrow A$



$A \leftrightarrow B$  $(A \rightarrow B) \wedge (B \rightarrow A)$ 

A	B	$A \leftrightarrow B$	$A \rightarrow B$	$B \rightarrow A$	$(A \rightarrow B) \wedge (B \rightarrow A)$
F	F	T	T	T	T
F	T	F	T	F	F
T	F	F	F	T	F
T	T	T	T	T	T

 $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$ 

$A \wedge B$

$\neg(\neg A \vee \neg B)$

A	B	$A \wedge B$	$\neg A$	$\neg B$	$\neg A \vee \neg B$	$\neg(\neg A \vee \neg B)$
F	F	F	T	T	T	F
F	T	F	T	F	T	F
T	F	F	F	T	T	F
T	T	T	F	F	F	T

$A \wedge B \equiv \neg(\neg A \vee \neg B)$



$$\begin{aligned} A \leftrightarrow B &\equiv (A \rightarrow B) \wedge (B \rightarrow A) \\ &\equiv (\neg A \vee B) \wedge (\neg B \vee A) \end{aligned}$$

○ Hukum De Morgan 1

$$\neg(A \wedge B) \equiv \neg A \vee \neg B$$

$$\neg\neg(A \wedge B) \equiv \neg(\neg A \vee \neg B)$$

$$A \wedge B \equiv \neg(\neg A \vee \neg B)$$

○ Hukum De Morgan 2

$$A \vee B \equiv \neg(\neg A \wedge \neg B)$$



$T = 1$

$F = 0$

A	1	0	$A^1$	$A^0$
F	T	F	F	F
T	T	F	T	F

- $A^1 \equiv A$  Identify of  $\wedge$
- $A^0 \equiv 0$  Zero of  $\wedge$
- $A \vee 1 \equiv 1$  Identify of  $\vee$
- $A \vee 0 \equiv A$  Zero of  $\vee$







### ○ Identity Laws

$$A \wedge 1 \equiv A$$

$$A \vee 0 \equiv A$$

### ○ Dominition Laws

$$A \vee 1 \equiv 1$$

$$A \wedge 0 \equiv 0$$

### ○ Tautology

$$A \vee \neg A \equiv 1$$

### ○ Contradiction

$$A \wedge \neg A \equiv 0$$

### ○ Idempotence Laws

$$A \vee A \equiv A$$

$$A \wedge A \equiv A$$

### ○ Law of Double Negation

$$\neg \neg A \equiv A$$

### ○ Commutative Laws

$$A \wedge B \equiv B \wedge A$$

$$A \vee B \equiv B \vee A$$

### ○ Assosiative Laws

$$(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$$

$$(A \vee B) \vee C \equiv A \vee (B \vee C)$$





## ○ Distributive Laws

$$A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$$

$$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$$

$$A \rightarrow B \equiv \neg A \vee B$$

$$A \rightarrow B \equiv \neg(A \wedge \neg B)$$

## ○ De Morgan's Law

$$\neg(A \wedge B) \equiv \neg A \vee \neg B$$

$$\neg(A \vee B) \equiv \neg A \wedge \neg B$$

$$A \leftrightarrow B \equiv (A \wedge B) \vee (\neg A \wedge \neg B)$$

$$A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$$



Identitas	$p \wedge \mathbf{1} \equiv p$	$p \vee \mathbf{0} \equiv p$
Ikatan	$p \vee \mathbf{1} \equiv \mathbf{T}$	$p \wedge \mathbf{0} \equiv \mathbf{0}$
Idempoten	$p \vee p \equiv p$	$p \wedge p \equiv p$
Negasi	$p \vee \neg p \equiv \mathbf{1}$	$p \wedge \neg p \equiv \mathbf{0}$
Negasi Ganda	$\neg \neg p \equiv p$	
Komutatif	$p \vee q \equiv q \vee p$	$p \wedge q \equiv q \wedge p$
Asosiatif	$(p \vee q) \vee r \equiv p \vee (q \vee r)$	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
Distributif	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
De Morgan's	$\neg(p \wedge q) \equiv \neg p \vee \neg q$	$\neg(p \vee q) \equiv \neg p \wedge \neg q$
Absorpsi	$p \wedge (p \vee q) \equiv p$	$p \vee (p \wedge q) \equiv p$

# BUKTIKAN BAHWA EKUIVALEN

1.  $A \rightarrow (\neg A \rightarrow B) \equiv 1$
2.  $(A \vee \neg B) \rightarrow C \equiv (\neg A \wedge B) \vee C$
3.  $A \rightarrow B \equiv \neg(A \wedge \neg B)$
4.  $\neg(\neg(A \wedge B) \vee B) \equiv 0$
5.  $\neg(P \vee \neg Q) \vee (\neg P \wedge \neg Q) \equiv \neg P$

